Nonperturbative and spin effects in the central exclusive production of tensor $\chi_c(2^+)$ meson

R. S. Pasechnik*

High Energy Physics, Department of Physics and Astronomy, Uppsala University Box 535, SE-75121 Uppsala, Sweden

A. Szczurek†

Institute of Nuclear Physics PAN, PL-31-342 Cracow, Poland and University of Rzeszów, PL-35-959 Rzeszów, Poland

O. V. Teryaev‡

Bogoliubov Laboratory of Theoretical Physics, JINR, Dubna 141980, Russia

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Abstract

We discuss central exclusive production (CEP) of the tensor $\chi_c(2^+)$ meson in proton-(anti)proton collisions at Tevatron, RHIC and LHC energies. The amplitude for the process is derived within the k_t-factorisation approach. Differential and total cross sections are calculated for several unintegrated gluon distributions (UGDFs). We compare exclusive production of all charmonium states $\chi_c(0^+)$, $\chi_c(1^+)$ and $\chi_c(2^+)$. Equally good description of the recent Tevatron data is achieved both with Martin-Ryskin phenomenological UGDF and UGDF based on unified BFKL-DGLAP approach. Unlike for Higgs production, the main contribution to the diffractive amplitude of heavy quarkonia comes from nonperturbative region of gluon transverse momenta $Q_\perp < 1\,\mathrm{GeV}$. At $y \approx 0$, depending on UGDF we predict the contribution of $\chi_c(1^+, 2^+)$ to the $J/\Psi + \gamma$ channel to be comparable or larger than that of the $\chi_c(0^+)$ one. This is partially due to a significant contribution from lower polarization states $\lambda = 0$ for $\chi_c(1^+)$ and $\lambda = 0, \pm1$ for $\chi_c(2^+)$ meson. Corresponding theoretical uncertainties are discussed.

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*Electronic address: roman.pasechnik@fysast.uu.se
†Electronic address: antoni.szczurek@ifj.edu.pl
‡Electronic address: teryaev@theor.jinr.ru

I. INTRODUCTION

It is well known that the exclusive diffractive Higgs production provides a very convenient tool for Higgs searches at hadron colliders due to a very clean environment unlike the inclusive production [1]. A QCD mechanism for the diffractive production of heavy central system has been proposed by Kaidalov, Khoze, Martin and Ryskin (Durham group) for Higgs production at the LHC (see Refs. [1–3]). Below we will refer to it as the KKMR approach. In the framework of this approach the amplitude of the exclusive $pp \to pXp$ process is considered to be a convolution of the hard subprocess amplitude describing fusion of two off-shell gluons producing a heavy system $g^*g^* \to X$, and the soft hadronic factors containing information about emission of the relatively soft gluons from the proton lines (see Fig. 1). In the framework of the $k_\perp$-factorisation approach these soft parts are written in terms of so-called off-diagonal unintegrated gluon distributions (UGDFs). The QCD factorisation is rigorously justified in the limit of very large factorisation scale being the transverse mass of the central system $M_\perp$.

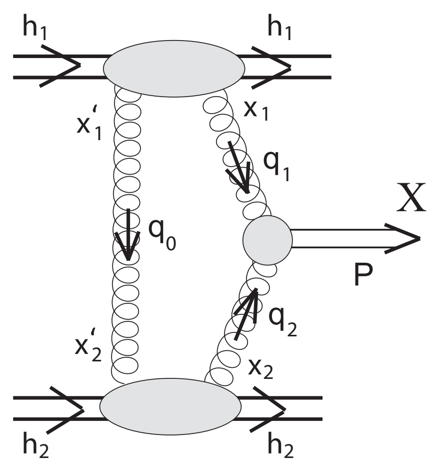

FIG. 1: *The QCD mechanism of diffractive production of the heavy central system X.*

In order to check the underlying production mechanism it is worth to replace the Higgs boson by a lighter (but still heavy enough to provide the QCD factorisation) meson which is easier to measure. In this respect the exclusive production of heavy quarkonia is under special interest from both experimental and theoretical point of view [4]. Verifying the KKMR approach against various data on exclusive meson production at high energies is a good test of nonperturbative dynamics of parton distributions encoded in UGDFs.

Recently, the signal from the diffractive $\chi_c(0^+, 1^+, 2^+)$ charmonia production in the radiative $J/\Psi + \gamma$ decay channel has been measured by the CDF Collaboration [5]: $d\sigma/dy|_{y=0}(pp \to pp(J/\psi + \gamma)) \simeq (0.97 \pm 0.26)$ nb. Assuming the absolute dominance of the spin-0 contribution, this result was published by the CDF Collaboration in the form:

$$\frac{d\sigma}{dy}\Big|_{y=0}(\chi_c(0^+)) \simeq \frac{1}{\mathrm{BR}(\chi_c(0^+) \to J/\psi + \gamma)}\frac{d\sigma}{dy}\Big|_{y=0}(pp \to pp(J/\psi + \gamma)) = (76 \pm 14)\,\mathrm{nb}.$$

Indeed, in the very forward limit the contributions from $\chi_c(1^+, 2^+)$ vanish due to the $J_z = 0$ selection rule [11, 27]. This is not true, however, for general kinematics [6, 7]. In particular, it was shown in Ref. [8] that the axial-vector $\chi_c(1^+)$ production, due to a relatively large

branching fraction of its radiative decay, may not be negligible and gives a significant contribution to the total signal measured by the CDF Collaboration. The same holds also for the tensor $\chi_c(2^+)$ meson contribution [9]. Recent Durham group investigations [10] support these predictions.

The production of the axial-vector $\chi_c(1^+)$ meson is additionally suppressed w.r.t. $\chi_c(0^+,2^+)$ in the limit of on-shell fusing gluons (with non-forward protons) due to the Landau-Yang theorem [8]. Such an extra suppression may, in principle, lead to the dominance of the $\chi_c(2^+)$ contribution over the $\chi_c(1^+)$ one in the radiative decay channel [9]. Off-shell effects play a significant role even for the scalar $\chi_c(0^+)$ production reducing the total cross section by a factor of $2-5$ depending on UGDFs [6]. The major part of the amplitude comes from rather small gluon transverse momenta $Q_\perp < 1\,\mathrm{GeV}$. This requires a special attention and including all polarisation states $\chi_c(1^+,2^+)$. Our present goal is to analyze these issues in more detail in the case of tensor charmonium production at the Tevatron, to study its energy dependence and to compare with corresponding contributions from scalar and axial-vector charmonia.

The paper is organized as follows. Section 2 contains the generalities of the QCD central exclusive production mechanism, two different prescriptions for off-diagonal UGDFs are introduced and discussed. In Section 3 we derive the hard subprocess amplitude $g^*g^* \to \chi_c(2^+)$ in the nonrelativistic QCD formalism and consider its properties. Section 4 contains numerical results for total and differential cross sections of $\chi_c(0^+,1^+,2^+)$ CEP and their correspondence to the last CDF data. In Section 5 the summary of main results is given.

II. DIFFRACTIVE $pp \to pp\chi_c(2^+)$ PRODUCTION AMPLITUDE

The general kinematics of the central exclusive production (CEP) process $pp \to pXp$ with X being the colour singlet $q\bar{q}$ bound state has already been discussed in our previous papers on $\chi_c(0^+)$ [6] and $\chi_c(1^+)$ [8] production. In this section we adopt the same notations and consider the matrix element for exclusive $\chi_c(2^+)$ production and its properties in detail.

According to the KKMR approach the amplitude of the exclusive double diffractive color singlet production $pp \to pp\chi_{cJ}$ is [6, 11]

$$\mathcal{M}^{pp\to pp\chi_{cJ}}_{J,\lambda} = s\cdot\pi^2\frac{1}{2}\frac{\delta_{c_1c_2}}{N_c^2-1}\int d^2q_{0,t}V^{c_1c_2}_{J,\lambda}(q_1,q_2,p_M)$$
$$\times\frac{f^{\rm off}_{g,1}(x_1,x_1',q_{0,t}^2,q_{1,t}^2,t_1)f^{\rm off}_{g,2}(x_2,x_2',q_{0,t}^2,q_{2,t}^2,t_2)}{q_{0,t}^2\,q_{1,t}^2\,q_{2,t}^2}\,, \tag{2.1}$$

where $t_{1,2}$ are the momentum transfers along the proton lines, q_0 is the momentum of the screening gluon, $q_{1,2}$ are the momenta of fusing gluons, and $f^{\rm off}_{g,i}(x_i,x_i',q_{0,t}^2,q_{i,t}^2,t_i)$ are the off-diagonal UGDFs (see Fig. 1).

Traditional (asymmetric) form of the off-diagonal UGDFs is taken in the limit of very small $x' \ll x_{1,2}$ in analogy to collinear off-diagonal gluon distributions (with factorized t-dependence) [12, 13], i.e.

$$\begin{aligned} f^{\rm off}_{g,1} &= R_g\,f^{(1)}_g(x_1,Q^{\rm eff\,2}_{1,t},\mu^2)\cdot F_N(t_1)\,,\\ f^{\rm off}_{g,2} &= R_g\,f^{(2)}_g(x_2,Q^{\rm eff\,2}_{2,t},\mu^2)\cdot F_N(t_2),\quad \mu^2=\frac{M_\perp^2}{4} \end{aligned} \tag{2.2}$$

with a quasiconstant prefactor R_g which accounts for the single $\log Q^2$ skewed effect [14] and is found to be 1.4 at the Tevatron energy and 1.2 at the LHC energy (for LO PDF), $Q^{\mathrm{eff}\,2}_{1/2,t} = \min(q_{0,t}^2, q_{1/2,t}^2)$ are the effective gluon transverse momenta, as adopted in Ref. [1, 11], $F_N(t)$ is the proton vertex factor, which can be parameterized as $F_N(t) = \exp(b_0 t)$ with $b_0 = 2\,\mathrm{GeV}^{-2}$ [15], or by the isoscalar nucleon form factor $F_1(t)$ as we have done in Ref. [6]. Below we shall refer to Eq. (2.2) as KMR UGDF[1].

Our results in Ref. [6] showed up a strong sensitivity of the KMRS numerical results [11] on the definition of the effective gluon transverse momenta $Q^{\mathrm{eff}}_{1/2,t}$ and the factorisation scales $\mu_{1,2}$. This behavior is explained by the fact that for χ_c production the great part of the diffractive amplitude (2.1) comes from nonperturbatively small $q_{0,t} < 1\,\mathrm{GeV}$. It means that the total diffractive process is dominated by very soft screening gluon exchanges with no hard scale and extremely small $x' \ll x_{1,2}$.

In principle, the factor R_g in Eq. (2.2) should be a function of x' and x_1 or x_2. In this case the off-diagonal UGDFs do not depend on x' and $q_{0,t}^2$ (or $q_{1/2,t}^2$), and their evolution is reduced to diagonal UGDFs evolution corresponding to one "effective" gluon. In general, the factor R_g can depend on UGDF and reflects complicated and still not well known dynamics in the small-x region.

In order to test this small-x dynamics and estimate the theoretical uncertainties related to introducing one "effective" gluon transverse momentum instead of two ones in Eq. (2.2), in Refs. [6, 16] we have used more generalized symmetrical prescription for the off-diagonal UGDFs. Actually, it is possible to calculate the off-diagonal UGDFs in terms of their diagonal counterparts as follows[2]

$$\begin{aligned} f^{\mathrm{off}}_{g,1} &= \sqrt{f_g^{(1)}(x_1', q_{0,t}^2, \mu_0^2) \cdot f_g^{(1)}(x_1, q_{1,t}^2, \mu^2)} \cdot F_N(t_1)\,, \\ f^{\mathrm{off}}_{g,2} &= \sqrt{f_g^{(2)}(x_2', q_{0,t}^2, \mu_0^2) \cdot f_g^{(2)}(x_2, q_{2,t}^2, \mu^2)} \cdot F_N(t_2)\,, \end{aligned} \tag{2.3}$$

where

$$x_1' = x_2', \quad \mu_0^2 = q_{0,t}^2, \quad \mu^2 = \frac{M_\perp^2}{4}.$$

This form of skewed two-gluon UGDFs (2.3) is inspired by the positivity constraints for the collinear Generalized Parton Distributions [17], and can be considered as a saturation of the Cauchy-Schwarz inequality for the density matrix [18]. It allows us to incorporate the actual dependence of the off-diagonal UGDFs on longitudinal momentum fraction of the soft screening gluon x' and its transverse momentum $q_{0,t}^2$ in explicitly symmetric way. As will be shown below, these symmetric off-diagonal UGDFs lead to results which are consistent with the Tevatron data.

However, trying to incorporate the actual dependence of UGDFs on (small but nevertheless finite) x' we may encounter a problem. The kinematics of the double diffractive process $pp \to pXp$ does not give any precise expression for x' in terms of the phase space integration variables. From the QCD mechanism under consideration one can only expect the general

[1] In actual calculations we use a more precise phenomenological Martin-Ryskin UGDF introduced in Ref. [13]. We are very thankful to L. Harland-Lang for a discussion on this point.

[2] For diagonal distributions without explicit scale dependences the μ_0^2, μ^2 arguments must be omitted.

inequality $x' \ll x_{1,2}$ and upper bound $x' \lesssim q_{0,t}/\sqrt{s}$ since the only scale appearing in the left part of the gluon ladder is the transverse momentum of the soft screening gluon $q_{0,t}$.

To explore the sensitivity of the final results on the values of x', staying in the framework of traditional KKMR approach, one can introduce naively $x' = \xi \cdot q_{0,t}/\sqrt{s}$ with an auxiliary parameter ξ [8]. In our earlier papers [6, 16] we considered the limiting case of maximal x' (with $\xi = 1$). However, it is worth to compare the predictions of the underlying QCD mechanism for smaller ξ against the available experimental data in order to estimate typical x' values. We will analyze this issue in greater detail in the Results section.

III. HARD SUBPROCESS $g^*g^* \to \chi_c(2^+)$ AMPLITUDE

Projection of the hard amplitude onto the singlet charmonium bound state $V_{\mu\nu}^{c_1c_2}$ is given by an 4-dimentional integral over relative momentum of quark and antiquark $q = (k_1 - k_2)/2$ [19, 20]:

$$V_{J,\,\mu\nu}^{c_1c_2}(k_1,k_2) = \mathcal{P}(q\bar{q} \to \chi_{cJ}) \bullet \Psi_{ik,\,\mu\nu}^{c_1c_2}(k_1,k_2) = 2\pi \cdot \sum_{i,k}\sum_{L_z,S_z}\frac{1}{\sqrt{m}}\int\frac{d^4q}{(2\pi)^4}\delta\left(q^0 - \frac{\mathbf{q}^2}{M}\right) \times$$
$$\times\, \Phi_{L=1,L_z}(\mathbf{q}) \cdot \langle L=1,L_z;S=1,S_z|J,J_z\rangle\,\langle 3i,\bar{3}k|1\rangle\;\mathrm{Tr}\left\{\Psi_{ik,\,\mu\nu}^{c_1c_2}\mathcal{P}_{S=1,S_z}\right\}, \quad (3.1)$$
$$\Psi_{ik,\,\mu\nu}^{c_1c_2} = -g^2\left[t_{ij}^{c_1}t_{jk}^{c_2}\cdot\left\{\gamma_\nu\frac{\hat{q}_{1,t}-\hat{k}_{1,t}-m}{(q_1-k_1)^2-m^2}\gamma_\mu\right\} - t_{kj}^{c_2}t_{ji}^{c_1}\cdot\left\{\gamma_\mu\frac{\hat{q}_{1,t}-\hat{k}_{2,t}+m}{(q_1-k_2)^2-m^2}\gamma_\nu\right\}\right].$$

Here the function $\Phi_{L=1,L_z}(\mathbf{q})$ is the momentum space wave function of the charmonium, the Clebsch-Gordan coefficient in color space is $\langle 3i,\bar{3}k|1\rangle = \delta^{ik}/\sqrt{N_c}$, the trace of t-matrices is $\mathrm{Tr}\,(t^{c_1}t^{c_2}) = \delta^{c_1c_2}/2$, and the projection operator $\mathcal{P}_{S=1,S_z}$ for a small relative momentum q has the form

$$\mathcal{P}_{S=1,S_z} = \frac{1}{2m}(\hat{k}_2 - m)\frac{\hat{\epsilon}(S_z)}{\sqrt{2}}(\hat{k}_1 + m). \quad (3.2)$$

Since P-wave function $\Phi_{L=1,L_z}$ vanishes at the origin, we may expand the trace in Eq. (3.1) in the Taylor series around $\mathbf{q} = 0$, and only the linear terms in q^σ survive. This yields an expression proportional to

$$\int\frac{d^3\mathbf{q}}{(2\pi)^3}q^\sigma\Phi_{L=1,L_z}(\mathbf{q}) = -i\sqrt{\frac{3}{4\pi}}\epsilon^\sigma(L_z)\mathcal{R}'(0), \quad (3.3)$$

with the derivative of the P-wave radial wave function at the origin $\mathcal{R}'(0)$ whose numerical value can be found in Ref. [21]. The general P-wave result (3.1) may be further reduced by employing the Clebsch-Gordan identity which for the tensor $\chi_{cJ=2}$ charmonium states reads

$$\mathcal{T}_{J=2}^{\sigma\rho} \equiv \sum_{L_z,S_z}\langle 1,L_z;1,S_z|2,J_z\rangle\,\epsilon^\sigma(L_z)\epsilon^\rho(S_z) = \epsilon^{\sigma\rho}(J_z).$$

Taking into account standard definitions of the light-cone vectors $n^+ = p_2/E_{cms},\ n^- = p_1/E_{cms}$ and momentum decompositions $q_1 = x_1p_1 + q_{1,t},\ q_2 = x_2p_2 + q_{2,t}$ and using the gauge invariance property (Gribov's trick) one gets the following projection (for any spin J)

$$q_1^\nu V_{J,\,\mu\nu}^{c_1c_2} = q_2^\mu V_{J,\,\mu\nu}^{c_1c_2} = 0,$$
$$V_J^{c_1c_2}(q_1,q_2) \;=\; n_\mu^+n_\nu^-V_{J,\,\mu\nu}^{c_1c_2}(q_1,q_2) = \frac{4}{s}\frac{q_{1,t}^\nu}{x_1}\frac{q_{2,t}^\mu}{x_2}V_{J,\,\mu\nu}^{c_1c_2}(q_1,q_2). \quad (3.4)$$

Since we adopt here the definition of the polarization vectors proportional to gluon transverse momenta $q_{1/2,t}$, then

$$V_{J,\lambda}^{c_1,c_2}(q_{1,t},q_{2,t}) \to 0, \quad q_{1,t} \to 0, \quad \text{or} \quad q_{2,t} \to 0\,. \tag{3.5}$$

It shows that gluon transverse momenta are necessary to get a nonzeroth diffractive cross section.

Summarizing all ingredients above, we get the vertex factor $g^*g^* \to \chi_c(2^+)$ in the following covariant form

$$\begin{aligned}
V_{J=2}^{c_1c_2} &= 2ig^2\sqrt{\frac{3}{M\pi N_c}}\frac{\delta^{c_1c_2}\mathcal{R}'(0)\epsilon_{\rho\sigma}^{(\lambda)}}{MM_\perp^2(q_1q_2)^2}\Big[(q_{1,t}q_{2,t})(q_1^\sigma-q_2^\sigma)\Big\{P^\rho(q_{1,t}^2-q_{2,t}^2)+(x_1p_1^\rho-x_2p_2^\rho)M^2- \\
&(q_{1,t}^\rho-q_{2,t}^\rho)M^2\Big\}-2(q_1q_2)\Big\{M^2(q_{1,t}^\rho q_{2,t}^\sigma+q_{1,t}^\sigma q_{2,t}^\rho)-q_{1,t}^2(q_{1,t}^\rho q_{2,t}^\sigma+q_{2,t}^\sigma q_{2,t}^\rho)- \\
&q_{2,t}^2(q_{1,t}^\sigma q_{2,t}^\rho+q_{1,t}^\sigma q_{1,t}^\rho)+(x_1p_1^\sigma-x_2p_2^\sigma)(q_{1,t}^2q_{2,t}^\rho-q_{2,t}^2q_{1,t}^\rho)+(q_{1,t}q_{2,t})(x_1p_1^\rho-x_2p_2^\rho)(q_{1,t}^\sigma-q_{2,t}^\sigma)- \\
&2q_{1,t}^2x_1p_1^\rho q_{2,t}^\sigma-2q_{2,t}^2x_2p_2^\rho q_{1,t}^\sigma+2(q_{1,t}q_{2,t})(x_1p_1^\sigma q_{2,t}^\rho+x_2p_2^\sigma q_{1,t}^\rho)+\frac{M_\perp^2}{s}(q_{1,t}q_{2,t})(p_1^\rho p_2^\sigma+p_2^\rho p_1^\sigma)\Big\}
\end{aligned} \tag{3.6}$$

Polarization tensor of $\chi_{cJ=2}$ satisfies the following relations (see e.g. Ref. [22])

$$P^\mu\epsilon_{\mu\nu}(\lambda)=P^\nu\epsilon_{\mu\nu}(\lambda)=0, \qquad \epsilon_{\mu\nu}(\lambda)=\epsilon_{\nu\mu}(\lambda), \qquad \epsilon_{\mu\mu}(\lambda)=0, \qquad \epsilon_{\mu\nu}(\lambda)\epsilon^{\mu\nu*}(\lambda')=\delta_{\lambda\lambda'},$$

$$\sum_{\lambda=0,\pm1,\pm2}\epsilon_{\mu\nu}(\lambda)\epsilon_{\rho\sigma}{}^*(\lambda)=\frac{1}{2}M_{\mu\rho}M_{\nu\sigma}+\frac{1}{2}M_{\mu\sigma}M_{\nu\rho}-\frac{1}{3}M_{\mu\nu}M_{\rho\sigma}, \qquad M_{\mu\nu}=g_{\mu\nu}-\frac{P_\mu P_\nu}{M^2}$$

One can check that it may be represented in the following general form

$$\begin{aligned}
\epsilon_{\mu\nu}(\pm|\lambda|) \;=\; & \frac{\sqrt{6}}{2}\delta_{0|\lambda|}\left(n_3^\mu n_3^\nu+\frac{1}{3}\left[g_{\mu\nu}-\frac{P_\mu P_\nu}{M^2}\right]\right) \\
&+\frac{1}{2}\delta_{1|\lambda|}\Big(i[n_2^\mu n_3^\nu+n_3^\mu n_2^\nu]\pm[n_1^\mu n_3^\nu+n_3^\mu n_1^\nu]\Big) \\
&-\frac{1}{2}\delta_{2|\lambda|}\Big(i[n_1^\mu n_2^\nu+n_2^\mu n_1^\nu]\pm[n_1^\mu n_1^\nu-n_2^\mu n_2^\nu]\Big)\,,
\end{aligned} \tag{3.7}$$

where $n_{1,2,3}$ are light-like basis vectors satisfying $n_\alpha^\mu n_\beta^\nu g_{\mu\nu}=g_{\alpha\beta}$ (with $n_0^\mu=P_\mu/M$), and $\lambda=0,\pm1,\pm2$ are the $\chi_c(2^+)$ meson helicities. To our best knowledge, there is no explicit decomposition of the meson polarisation tensor $\epsilon_{\mu\nu}(\lambda)$ in terms of basis vectors n_i like Eq. (3.7) in the literature. In practical calculations below it is convenient to use it in a different representation:

$$\begin{aligned}
\epsilon_{\mu\nu}(\lambda) \;=\; & \frac{\sqrt{6}}{12}(2-|\lambda|)(1-|\lambda|)\left[g_{\mu\nu}-\frac{P_\mu P_\nu}{M^2}\right]+\frac{\sqrt{6}}{4}(2-|\lambda|)(1-|\lambda|)n_3^\mu n_3^\nu+ \\
&+\frac{1}{4}\lambda(1-|\lambda|)[n_1^\mu n_1^\nu-n_2^\mu n_2^\nu]+\frac{1}{4}i|\lambda|(1-|\lambda|)[n_1^\mu n_2^\nu+n_2^\mu n_1^\nu]+ \\
&+\frac{1}{2}\lambda(2-|\lambda|)[n_1^\mu n_3^\nu+n_3^\mu n_1^\nu]+\frac{1}{2}i|\lambda|(2-|\lambda|)[n_2^\mu n_3^\nu+n_3^\mu n_2^\nu]\,.
\end{aligned}$$

Similarly to what has been done for $\chi_c(1^+)$ production in Ref. [8], in the c.m.s. frame we choose the basis with collinear $\mathbf{n}_3$ and $\mathbf{P}$ vectors (so, we have $\mathbf{P}=(E,0,0,P_z),\ P_z=|\mathbf{P}|>0$) as a simplest one

$$n_1^\beta=(0,\,1,\,0,\,0),\ \ n_2^\beta=(0,\,0,\,1,\,0),\ \ n_3^\beta=\frac{1}{M}\,(|\mathbf{P}|,0,0,E),\ \ |\mathbf{P}|=\sqrt{E^2-M^2}. \tag{3.8}$$

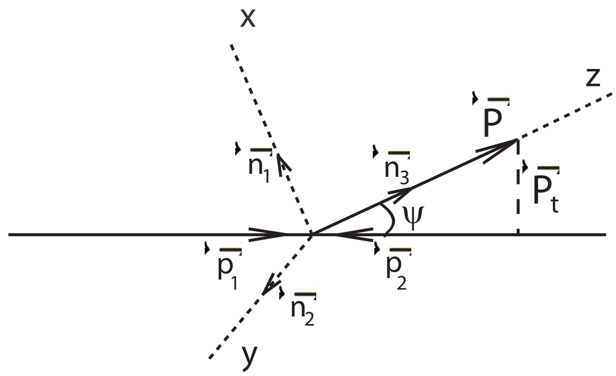

FIG. 2: *Coordinate basis in the center-of-mass system of incoming protons $p_{1,2}$.*

Note, that we choose $\mathbf{n}_2$ to be transverse to the c.m.s beam axis (see Fig. 2), while $\mathbf{n}_1$, $\mathbf{n}_3$ are turned around by the polar angle $\psi = [0 \ldots \pi]$ between $\mathbf{P}$ and the c.m.s. beam axis. In the considered basis $\{\mathbf{n}_1,\, \mathbf{n}_2,\, \mathbf{n}_3\}$ we have the following coordinates of the incoming protons

$$p_1 = \frac{\sqrt{s}}{2}(1,\, -\sin\psi,\, 0,\, \cos\psi), \quad p_2 = \frac{\sqrt{s}}{2}(1,\, \sin\psi,\, 0,\, -\cos\psi)\ . \tag{3.9}$$

The gluon transverse momenta with respect to the c.m.s. beam axis are

$$q_{1,t} = (0,\, Q^x_{1,t}\cos\psi,\, Q^y_t,\, Q^x_{1,t}\sin\psi), \quad q_{2,t} = (0,\, Q^x_{2,t}\cos\psi,\, -Q^y_t,\, Q^x_{2,t}\sin\psi),$$

where $Q^x_{1/2,t}$, $\pm Q^y_t$ are the components of the gluon transverse momenta in the basis with the z-axis collinear to the c.m.s. beam axis.

From definition (3.9) it follows that energy of the meson and polar angle ψ are related to covariant scalar products in the considered coordinate system as [8]

$$E = \frac{(p_1P)+(p_2P)}{\sqrt{s}}, \quad \cos\psi = \frac{(p_1P)-(p_2P)}{\sqrt{s}|\mathbf{P}|}, \quad \sin\psi = \frac{(p_2n_1)-(p_1n_1)}{\sqrt{s}}. \tag{3.10}$$

Furthermore, we also see that from $q_1 = x_1p_1 + q_{1,t}$, $q_2 = x_2p_2 + q_{2,t}$ and $q_1 + q_2 = P$ we have

$$x_1 = \frac{E + |\mathbf{P}|\cos\psi}{\sqrt{s}}, \qquad x_2 = \frac{E - |\mathbf{P}|\cos\psi}{\sqrt{s}}\ . \tag{3.11}$$

Relations (3.10) and (3.11) show that the interchange of proton momenta $p_1 \leftrightarrow p_2$ is equivalent to the interchange of the angle $\psi \leftrightarrow \psi \pm \pi$, i.e. $\sin\psi \leftrightarrow -\sin\psi$ and $\cos\psi \leftrightarrow -\cos\psi$ simultaneously. The last permutation also provides the interchange of the longitudinal components of gluons momenta $x_1 \leftrightarrow x_2$.

Conservation laws provide us with the following relations between components of gluon transverse momenta and covariant scalar products

$$Q^x_{1,t} = -\frac{q^2_{1,t} + (q_{1,t}q_{2,t})}{|\mathbf{P}|\sin\psi}, \quad Q^x_{2,t} = -\frac{q^2_{2,t} + (q_{1,t}q_{2,t})}{|\mathbf{P}|\sin\psi}, \quad Q^y_t = \frac{\sqrt{q^2_{1,t}q^2_{2,t} - (q_{1,t}q_{2,t})^2}}{|\mathbf{P}_t|}\,\mathrm{sign}(Q^y_t),$$
$$P^2_t = -|\mathbf{P}_t|^2 = -|\mathbf{P}|^2\sin^2\psi = q^2_{1,t} + q^2_{2,t} + 2(q_{1,t}q_{2,t}), \quad q^2_{1/2,t} = -|\mathbf{q}_{1/2,t}|^2,$$

where $|\mathbf{P}_t| = |\mathbf{P}||\sin\psi|$ is the meson transverse momentum with respect to the z-axis. The appearance of the factor $\mathrm{sign}(Q_t^y)$ guarantees the applicability of (3.12) for positive and negative Q_t^y. Note that under permutations $q_{1,t} \leftrightarrow q_{2,t}$ implied by Bose statistics the components interchange as $Q_{1,t}^x \leftrightarrow Q_{2,t}^x$ and $Q_t^y \leftrightarrow -Q_t^y$. In our notations the quantity $\sin\psi$ plays a role of the noncollinearity of meson in considered coordinates. A straightforward calculation leads to the following vertex function in these coordinates

$$\begin{aligned}
&V^{c_1c_2}_{J=2,\lambda} = 2ig^2\delta^{c_1c_2}\sqrt{\frac{1}{3M\pi N_c}}\frac{\mathcal{R}'(0)}{M|\mathbf{P}_t|^2(M^2-q_{1,t}^2-q_{2,t}^2)^2}\times \\
&\Big[6M^2\, i|\lambda|(q_{1,t}^2-q_{2,t}^2)\,\mathrm{sign}(Q_t^y)\Big\{|[\mathbf{q}_{1,t}\times\mathbf{q}_{2,t}]\times\mathbf{n}_1|\,(1-|\lambda|)\,\mathrm{sign}(\sin\psi)\,\mathrm{sign}(\cos\psi)+ \\
&2\,|[\mathbf{q}_{1,t}\times\mathbf{q}_{2,t}]\times\mathbf{n}_3|\,(2-|\lambda|)\Big\} - \big[2q_{1,t}^2q_{2,t}^2+(q_{1,t}^2+q_{2,t}^2)(q_{1,t}q_{2,t})\big]\Big\{3M^2(\cos^2\psi+1)\lambda(1-|\lambda|)+ \\
&6ME\sin(2\psi)\,\lambda(2-|\lambda|)\,\mathrm{sign}(\sin\psi)\,\mathrm{sign}(\cos\psi)+\sqrt{6}\,(M^2+2E^2)\sin^2\psi\,(1-|\lambda|)(2-|\lambda|)\Big\}\Big]\,,
\end{aligned} \tag{3.12}$$

where

$$\begin{aligned}
|[\mathbf{q}_{1,t}\times\mathbf{q}_{2,t}]\times\mathbf{n}_1| &= \sqrt{q_{1,t}^2q_{2,t}^2-(q_{1,t}q_{2,t})^2}\,|\cos\psi|, \\
|[\mathbf{q}_{1,t}\times\mathbf{q}_{2,t}]\times\mathbf{n}_3| &= \frac{E}{M}\sqrt{q_{1,t}^2q_{2,t}^2-(q_{1,t}q_{2,t})^2}\,|\sin\psi|.
\end{aligned}$$

The amplitude (3.12) explicitly obeys the Bose symmetry under the interchange of gluon momenta and polarizations due to resulting simultaneous permutations $\cos\psi \leftrightarrow -\cos\psi$, $\sin\psi \leftrightarrow -\sin\psi$ and $Q_t^y \leftrightarrow -Q_t^y$.

It follows from the conservation laws that

$$q_{1t}+p'_{1t} = -q_{0t}, \qquad q_{2t}+p'_{2t} = q_{0t}, \qquad P_t = -(p'_{1t}+p'_{2t})$$

Let us consider first the limit of the "coherent" scattering of protons $p'_{1t} = p'_{2t} \equiv p_t$, so

$$q_{1t} = -(p_t+q_{0t}), \qquad q_{2t} = -(p_t-q_{0t}), \qquad P_t = -2p_t, \qquad p_t^y = 0. \tag{3.13}$$

The production vertex (3.12) in this limit has the form

$$\begin{aligned}
&V^{c_1c_2}_{J=2,\lambda}(q_{0t}^x,q_{0t}^y,p_t) = 2ig^2\delta^{c_1c_2}\sqrt{\frac{1}{3M\pi N_c}}\frac{\mathcal{R}'(0)}{M(M^2-2(p_t^2+q_{0t}^2))^2}\times \\
&\Big[12M^2\, i|\lambda|\, q_{0t}^xq_{0t}^y\Big\{(1-|\lambda|)\,\cos\psi+\frac{2E}{M}\,(2-|\lambda|)\,\sin\psi\Big\}+ \\
&\big[p_t^2+(q_{0t}^x)^2-(q_{0t}^y)^2\big]\Big\{3M^2(\cos^2\psi+1)\lambda(1-|\lambda|)+ \\
&6ME\sin(2\psi)\,\lambda(2-|\lambda|)\,\mathrm{sign}(\sin\psi)\,\mathrm{sign}(\cos\psi)+\sqrt{6}\,(M^2+2E^2)\sin^2\psi\,(1-|\lambda|)(2-|\lambda|)\Big\}\Big]
\end{aligned} \tag{3.14}$$

We see that in contrast to the axial-vector case considered in Ref. [8], the diffractive amplitude of $\chi_c(2^+)$ production does not turn to zero in this "coherent" limit for $p_t \neq 0$.

In the forward limit $p_t \to 0$ (which is a particular case of the "coherent" one) the amplitude turns to zero at any meson rapidities y. Indeed, we have $P_t \to 0$ and $\sin\psi \to \pm 0$ and the amplitude turns into

$$V^{c_1c_2}_{J=2,\,\lambda}(q^x_{0t}, q^y_{0t}, p_t \to 0) \;=\; g^2\delta^{c_1c_2}\sqrt{\frac{1}{3M\pi N_c}}\frac{12M\,\mathcal{R}'(0)}{(M^2-2q^2_{0t})^2}\times \qquad (3.15)$$
$$(1-|\lambda|)\big\{i\lambda\big[(q^x_{0t})^2-(q^y_{0t})^2\big]-2\,|\lambda|\,q^x_{0t}q^y_{0t}\,\mathrm{sign}(\cos\psi)|_{\psi\to 0,\pi}\big\}\,.$$

The imaginary part of this vertex function turns out to be antisymmetric w.r.t. interchanging $q^x_{0t} \leftrightarrow q^y_{0t}$, whereas its real part is antisymmetric w.r.t. changing the sign of q^x_{0t} or q^y_{0t} component, i.e.

$$\Im V^{c_1c_2}_{J=2,\,\lambda}(q^x_{0t}, q^y_{0t}, p_t \to 0) = -\Im V^{c_1c_2}_{J=2,\,\lambda}(q^y_{0t}, q^x_{0t}, p_t \to 0)$$
$$\Re V^{c_1c_2}_{J=2,\,\lambda}(q^x_{0t}, q^y_{0t}, p_t \to 0) = -\Re V^{c_1c_2}_{J=2,\,\lambda}(-q^x_{0t}, q^y_{0t}, p_t \to 0) = -\Re V^{c_1c_2}_{J=2,\,\lambda}(q^x_{0t}, -q^y_{0t}, p_t \to 0)\,.$$

Since in this case $q_{1t}=-q_{0t}$, $q_{2t}=q_{0t}$ in the forward limit, then the double integral in the diffractive amplitude has an antisymmetric integrand and turns to zero in the symmetric limit

$$\mathcal{M}_{p_t\to 0} \sim F_1(t_1)F_1(t_2)\int dq^x_{0t}dq^y_{0t}\frac{V_{J=2}(q^x_{0t}, q^y_{0t}, p_t \to 0)\cdot f(x_1, q^2_{0,t}, q^2_{0,t})f(x_2, q^2_{0,t}, q^2_{0,t})}{q^6_{0t}} = 0. \qquad (3.16)$$

This explicitly confirms the observation made in Refs. [3, 23][3].

Very recently, when our paper was almost complete, a paper by L. Harland-Lang, V. Khoze, M. Ryskin and W. Stirling (HKRS) [10] appeared where the hard subprocess amplitudes $gg \to \chi_c(J^+)$ (based on formalism by Kuhn et al for $\gamma^*\gamma^* \to \chi_c(J^+)$ [24]) including the gluon virtualities were listed for different spins including the tensor $\chi_c(2^+)$:

$$V^{\rm HKRS}_{J=0} \;=\; \sqrt{\frac{1}{6}}\frac{c}{M}\big[3M^2(q_{1,t}q_{2,t})-(q_{1,t}q_{2,t})(q^2_{1,t}+q^2_{2,t})-2q^2_{1,t}q^2_{2,t}\big], \qquad (3.17)$$
$$V^{\rm HKRS}_{J=1,\lambda} \;=\; -\frac{2ic}{s}p_{1,\nu}p_{2,\alpha}\varepsilon^{\mu\nu\alpha\beta}\epsilon_\beta\big[(q_{2,t})_\mu q^2_{1,t}-(q_{1,t})_\mu q^2_{2,t}\big], \qquad (3.18)$$
$$V^{\rm HKRS}_{J=2,\lambda} \;=\; \frac{\sqrt{2}cM}{s}\,\epsilon^{\mu\alpha}\big[s(q_{1,t})_\mu(q_{2,t})_\alpha+2(q_{1,t}q_{2,t})p_{1,\mu}p_{2,\alpha}\big], \qquad (3.19)$$

where the constant prefactor is

$$c = \frac{1}{2\sqrt{N_c}}\frac{4g^2}{(q_1q_2)^2}\sqrt{\frac{6}{4\pi M}}\mathcal{R}'(0)\,.$$

The first amplitude $V^{\rm HKRS}_{J=0}$ (3.17) is the same as the expression obtained in Ref. [6] (up to a factor of 2 coming from different normalisations of the hard part $n^+_\mu n^-_\nu V_{J,\,\mu\nu}$ in our case and $(2/s)p^\mu_1 p^\nu_2 V_{J,\,\mu\nu}$ in Ref. [10]), where the major role of the gluon virtualities in the hard subprocess amplitude of quarkonia production was claimed to be crucial. In particular, it

[3] We are grateful to V. A. Khoze for helpful discussions of this problem.

was shown that an account of the gluon virtualities reduces the previous KMRS result in Ref. [11] for on-mass-shell gluons $V_0^{\rm KMRS} \sim (q_{1,t}q_{2,t})$ by a factor of $2-3$.

The second amplitude, $V_{J=1,\lambda}^{\rm HKRS}$, looks different from our previous result, obtained in Ref. [8]. However, one can directly check that the difference between the amplitudes (3.18) and (2.12) in Ref. [8] turns to zero when fixing the coordinates in the c.m.s. frame of reference as in Eq. (3.9) (see also Fig. 2) and the meson polarisation vector ϵ^β with the basis as in Eq. (3.8). Due to the covariant structure of these amplitudes, the last observation means that they are the same in any frame of reference. The calculations proving this equality are rather involved, and we do not show them explicitly here.

Very similar situation holds for $\chi_c(2^+)$ production amplitudes. Namely, the amplitudes (3.19) and (3.6) turned out to be the same under fixing the coordinates as in the previous section. Therefore, under the kinematical relations our results for the hard subprocess amplitudes are in complete agreement with the corresponding HKRS results. Let us now turn to the discussion of numerical results.

IV. NUMERICAL RESULTS

Results for the differential cross sections $d\sigma/dy(y=0)$ of the diffractive $\chi_c(0^+,1^+,2^+)$ meson production at the Tevatron energy $W=1960$ GeV for different UGDFs are shown in Table I. In the last column we show the results for the expected signal in the $J/\psi+\gamma$ channel summed over all χ_c spin states (and all polarisation states of $\chi_c(1^+,2^+)$ mesons)

$$\frac{d\sigma_{obs}}{dy}\Big|_{y=0} \approx \sum_{J=0,1,2} K_{\rm NLO}^{(J)}\langle S^2_{\rm eff}\rangle_J {\rm BR}(\chi_c(J^+)\to J/\Psi+\gamma)\frac{d\sigma^{bare}_{\chi_c(J^+)}}{dy}\Big|_{y=0}, \tag{4.1}$$

which can be compared with the corresponding value measured by the CDF Collaboration [5]: $d\sigma^{exp}/dy|_{y=0}(pp\to pp(J/\psi+\gamma))\simeq(0.97\pm0.26)$ nb.

In Refs. [10, 11] is was assumed that the NLO corrections factor $K_{\rm NLO}$ in the $g^*g^*\to\chi$ vertex is the same as in the $\chi\to gg$ width implying that $|V_J|^2\sim\Gamma(\chi\to gg)$. In general, such corrections depend on spin of $q\bar q$ resonance. So, the diffractive cross section for each $\chi_c(J^+)$ has to be multiplied by not necessarily the same factor $K_{\rm NLO}^{(J)}$, as shown in Eq. (4.1)[4]. This can be done, however, only for 0^+ and 2^+ states, and the corresponding NLO QCD radiative corrections are well-known [25]:

$$K_{\rm NLO}^{(0)} = 1+8.77\frac{\alpha_s(M_\chi)}{\pi}\simeq 1.68, \qquad K_{\rm NLO}^{(2)} = 1-4.827\frac{\alpha_s(M_\chi)}{\pi}\simeq 0.63\,. \tag{4.2}$$

Due to the Landau-Yang theorem the decay of the axial vector charmonium 1^{++} to on-shell gluons is forbidden, and there are no reliable calculations of the NLO QCD corrections to its coupling with off-shell gluons. In the following we take naively $K_{\rm NLO}^{(1)}=1$. This leads to an additional uncertainty of the model predictions.

As has been claimed in Refs. [7, 10] the absorptive corrections are quite sensitive to the meson spin-parity. This was studied before in the context of scalar and pseudoscalar Higgs

[4] See Ref. [10] for discussion of extra uncertainties coming from NNLO and higher order corrections.

production in Ref. [2]. We adopt here the following effective gap survival factors, calculated in Ref. [10] for different spins including eikonal and so-called enhanced contributions:

$$\langle S^2_{\rm eff}(\chi_c(0^+))\rangle \simeq 0.033, \qquad \langle S^2_{\rm eff}(\chi_c(1^+))\rangle \simeq 0.050, \qquad \langle S^2_{\rm eff}(\chi_c(2^+))\rangle \simeq 0.073\,. \quad (4.3)$$

The contribution of the scalar $\chi_c(0^+)$ CEP, which was initially assumed to be the dominant one [11], is reduced by a very small branching ratio of its observable radiative decay [8, 10]. In turn, the strong suppression of the $\chi_c(1^+)$ central production in both the on-mass-shell limit of fusing gluons (due to Landau-Yang theorem [26]) and the forward scattering limit of outgoing protons (due to the so-called $J_z = 0$ selection rule [11, 27]) may be partially compensated by its much higher branching ratio to the observed $J/\psi+\gamma$ final state [8]. Analogously to the axial-vector case, the suppression of the tensor $\chi_c(2^+)$ CEP is likely to be eliminated by its large decay branching ratio [10], and the resulting value of the radiative decay signal is under our special interest.

TABLE I: Differential cross section $d\sigma_{\chi_c}/dy(y = 0)$ (in nb) of the exclusive diffractive production of $\chi_c(0^+, 1^+, 2^+)$ mesons and their partial and total signal in radiative $J/\psi + \gamma$ decay channel $d\sigma_{J/\psi\gamma}/dy(y = 0)$ at Tevatron for different UGDFs, cuts on the transverse momentum of the gluons in the loop ($q_{0,t}$) and different values of the auxiliary parameter ξ controlling the characteristic x' values in the symmetric skewed UGDFs prescription (2.3) (denoted as "sqrt"). NLO skewedness factor $R_g^{NLO} = 1.3$ for the KMR asymmetric prescription (2.2) (denoted as "R_g"), NLO correction factors (4.2) and absorptive correction factors (4.3) are included. Contributions from all polarisations are incorporated.

skewed UGDF		$\chi_c(0^+)$		$\chi_c(1^+)$		$\chi_c(2^+)$		ratio		signal
prescription	ξ	$\frac{d\sigma_{\chi_c}}{dy}$	$\frac{d\sigma_{J/\psi\gamma}}{dy}$	$\frac{d\sigma_{\chi_c}}{dy}$	$\frac{d\sigma_{J/\psi\gamma}}{dy}$	$\frac{d\sigma_{\chi_c}}{dy}$	$\frac{d\sigma_{J/\psi\gamma}}{dy}$	$1^+/0^+$	$2^+/0^+$	$\frac{d\sigma_{obs}}{dy}$
GBW [30], "sqrt"	1.0	13.2	0.15	0.01	0.003	1.96	0.38	0.02	2.5	0.5
	0.3	12.8	0.15	0.04	0.01	1.39	0.27	0.07	1.8	0.4
lin KS [31], "R_g"	—	32.6	0.37	0.20	0.07	0.53	0.10	0.2	0.3	0.5
lin KS [31], "sqrt"	1.0	17.2	0.19	0.12	0.04	0.61	0.12	0.2	0.6	0.4
nlin KS [31], "sqrt"	1.0	12.6	0.14	0.07	0.02	0.39	0.08	0.1	0.6	0.3
	0.3	20.6	0.23	0.10	0.03	0.57	0.11	0.1	0.5	0.4
	0.05	**36.0**	**0.41**	**0.13**	**0.04**	**0.84**	**0.16**	**0.1**	**0.4**	**0.6**
KMR [13], GRV94HO, $(q_{0,t}^{cut})^2 = 0.72\,\mathrm{GeV}^2$	—	13.5	0.16	0.06	0.02	0.23	0.05	0.1	0.3	0.2
KMR [13], GRV94HO, $(q_{0,t}^{cut})^2 = 0.36\,\mathrm{GeV}^2$	—	**37.9**	**0.43**	**0.14**	**0.05**	**0.75**	**0.15**	**0.1**	**0.3**	**0.6**
HKRS result [10] $(q_{0,t}^{cut})^2 = 0.72\,\mathrm{GeV}^2$	—	27.1	0.31	0.72	0.25	0.95	0.19	0.8	0.6	0.7

As it was discussed in Ref. [6], the dominant contribution to the diffractive CEP of $\chi_c(0^+)$ comes from nonperturbative values of the gluon transverse momenta $q_t < 1\,\mathrm{GeV}$. In order to estimate the role of small q_t in the central production of $\chi_c(1^+, 2^+)$ and related theoretical

uncertainties we use different UGDFs known from the literature (for details see Refs. [6, 29]). Among them there are perturbatively modeled KMR UGDF [1, 12, 13], which include the Sudakov form factor, as well as GBW [30] and linear/nonlinear Kutak-Staśto (KS) [31] UGDF models which by construction can be used for any values of the gluon transverse momenta.

In the last row of Table I we show the HKRS results for partial cross sections extracted from their original paper [10]. These cross sections were calculated by the HKRS at some small energy scale and subsequently extrapolated up to the Tevatron energy assuming a Regge type energy dependence.

By direct calculation at the Tevatron energy with the same KMR UGDFs, but without imposing any arguments beyond the QCD framework (like Regge scaling, for example), we get the observable $J/\psi\gamma$ cross section $d\sigma_{J/\psi\gamma}/dy(y = 0) \simeq 0.6$ nb, which is close to HKRS result $\sim$ 0.7 nb, but somewhat lower than the present CDF result $d\sigma^{exp}/dy|_{y=0}(pp \to pp(J/\psi + \gamma)) \simeq (0.97 \pm 0.26)$ nb. However, this demanded to incorporate physics below HKRS cut-off on gluon transverse momentum in the loop integral (2.1) $(q_{0,t}^{cut})^2 = 0.72\,\mathrm{GeV}$ underlining the importance of nonperturbative contributions of small $q_{0,t}$ in the QCD mechanism under consideration. Relations between different χ_c's obtained in Ref. [10] are not reproduced as well. Our result is highlighted in bold in Table I.

The reason of such a discrepancy in both the normalization and relative contributions of different spins to the observable rate is the on-shell approximation $M_X^2 \gg q_{1/2,t}^2$ in the hard $g^*g^* \to \chi_c(J^+)$ adopted by HKRS[5]. As was firstly noticed in Ref. [6], the gluon off-shellness plays an important role in the central exclusive $\chi_c(0^+)$ leading to a strong reduction of the cross section depending on UGDF model. Now, we observe that the same effect significantly affects the ratios between different spin contributions.

Note that at zeroth meson rapidity $y = 0$ a significant part of the cross section comes from lower polarisation states in the center-of-mass frame $\lambda = 0$ ($\chi_c(1^+)$) and $\lambda = 0, \pm 1$ ($\chi_c(2^+)$). In the total (integrated over y) cross section the maximal helicity contributions, however, strongly dominate. We leave a more detailed investigation of the polarisation effects for a separate publication.

Relative contributions of $\chi_c(0^+, 1^+, 2^+)$ CEP to observable signal $(J/\Psi + \gamma)$ require an additional discussion. Last PDG updated set of branching ratios for charmonia radiative decays is [28]:

$$
\begin{aligned}
&\mathrm{BR}(\chi_c(0^+) \to J/\psi + \gamma) = 0.0114,\\
&\mathrm{BR}(\chi_c(1^+) \to J/\psi + \gamma) = 0.341,\\
&\mathrm{BR}(\chi_c(2^+) \to J/\psi + \gamma) = 0.194.
\end{aligned}
$$

Furthermore, as one can see in Table I, despite of larger branching ratio in the axial-vector case the observable signal from the $\chi_c(1^+)$ CEP occurs to be smaller than that from $\chi_c(2^+)$ for UGDFs enhanced at sufficiently small nonperturbative q_t (in particular, for the Kutak-Staśto (KS) and GBW UGDFs) due to an additional suppression of the $g^*g^* \to \chi_c(1^+)$ subprocess vertex at small $q_{1/2,t}$. For the GBW UGDF the $\chi_c(1^+)$ contribution is strongly suppressed whereas the $\chi_c(2^+)$ contribution turned out to be larger than the $\chi_c(0^+)$ one. All

[5] We are most thankful to L. Harland-Lang for the helpful discussions and correspondence on the topic. The exchange of the Fortran codes between us helped a lot in cross-checks of our calculations and understanding the various sources of discrepancies.

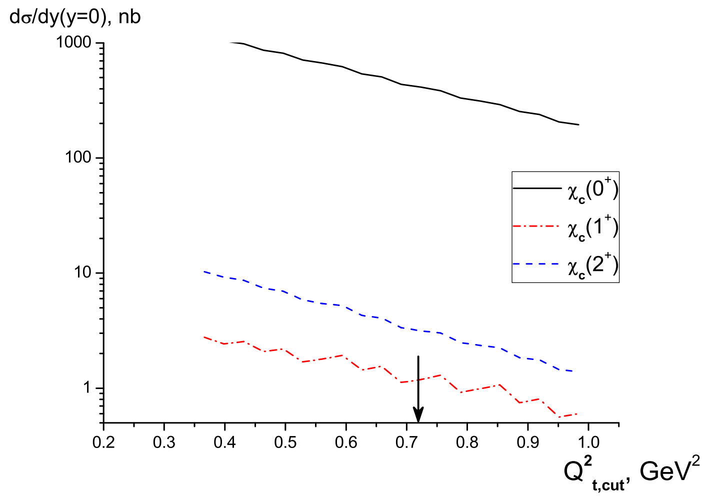

FIG. 3: *Dependence of the differential cross section* $d\sigma_{\chi_c}/dy(y=0)$ *of* $\chi_c(0^+,1^+,2^+)$ *CEP on the infrared cut-off on small effective gluon transverse momentum* Q_t^{cut} *for the KMR UGDF with GRV94HO* ($R_g = 1.3$)*. Absorption effects are not included here. Arrow points to the HKRS cut-off* $0.72\,GeV^2$ *[10].*

the UGDF models under considerations lead to somewhat underestimated observable signal at Tevatron $\lesssim$ 0.6 nb, however, it can be still reliable within relatively large theoretical uncertainties of the gap survival factors and QCD mechanism under discussion. In the case of the KS model, contributions from $\chi_c(1^+)$ state are found to be stronger suppressed than in KMR model. Measurements of separate spin contributions thus would help to distinguish between different UGDF models.

TABLE II: Integrated over full phase space (bare) cross sections (in nb) for the central exclusive $\chi_c(0^+,1^+,2^+)$ production at RHIC, Tevatron and LHC energies. Infrared cut-off for the KMR UGDF (in "R_g"-prescription) is taken to be $(Q_t^{cut})^2 = 0.72\,\mathrm{GeV}^2$. We take R_g to be equal 1.3 at all three energies. Absorption effects are not included here.

χ_c	UGDF	RHIC	Tevatron	LHC
$\chi_c(0^+)$	nlin. KS, $\xi = 0.3$	108	3569	23260
	KMR, GRV94HO	43	2270	30720
$\chi_c(1^+)$	nlin. KS, $\xi = 0.3$	0.4	12	65
	KMR, GRV94HO	0.2	7	87
$\chi_c(2^+)$	nlin. KS, $\xi = 0.3$	2	44	209
	KMR, GRV94HO	0.4	18	195

In the case of the KMR UGDF, we observe quite substantial dependence of the predicted observable signal w.r.t. variations of the infrared cut-off on small transverse momenta of the gluons in the most internal loop (see Fig. 3). From Table I we see that the shift of Q_t^{cut} from the value $0.72\,\mathrm{GeV}^2$ used in Ref. [10] down to the minimal perturbative scale of the

integrated GRV94HO distributions $0.36\,\mathrm{GeV}^2$ [32] leads to increase of the cross section by a factor of about 3, approaching the CDF data. For comparison, decrease of the Q_{cut} from $1\,\mathrm{GeV}^2$ down to $0.36\,\mathrm{GeV}^2$ leads to increase of the cross section by a factor of 6. Since we can not estimate the nonperturbative contribution coming from below $0.36\,\mathrm{GeV}^2$, this allows us to conclude that perturbatively motivated KMR UGDF leads to infrared unstable result in the case of relatively light charmonium CEP. It is clear that the essential part of the QCD dynamics comes from the nonperturbative region of transverse momenta below the HKRS cut-off $0.72\,\mathrm{GeV}^2$ [10]. KS and GBW UGDFs allow to incorporate some unknown physics even below the minimal GRV scale $Q_0^2 = 0.36\,\mathrm{GeV}^2$, avoiding ambiguities in defining the effective gluon momenta.

Applying the KMR's asymmetrical off-diagonal UGDF according to Eq. (2.2) ("R_g" prescription) in the case of the GBW models we get strongly overestimated observable signal at Tevatron, which means that in this case it is crucial to take into account the x'-dependence of off-diagonal UGDFs when going deeply into the infrared region of small q_t's. The x'-dependent "sqrt" prescription, introduced in Eq. (2.3), leads to observable signal, which is much closer to the experimental data.

The "sqrt" prescription, introduced in Eq. (2.3), provides an agreement with the data (within a factor of 2 in overall theoretical uncertainty between different UGDFs) with the KS (with rather small $\xi \sim 0.05$) and GBW models giving the cross section $d\sigma_{obs}/dy(y=0) \simeq 0.5-0.6$ (see Table I). This practically means that the smaller $q_{0,t}$ comes into the game, the smaller x' w.r.t. $q_{0,t}^2/s$ is required to get the data description, providing one more argument about importance of nonperturbative effects in charmonia CEP. The relative contributions of different charmonium states in the $J/\psi+\gamma$ channel (including absorption effects) in the case of, e.g., KS model are found to be:

$$\left(\frac{d\sigma^{\chi_{c0}}_{J/\psi\gamma}}{dy}\right)_{KS} : \left(\frac{d\sigma^{\chi_{c1}}_{J/\psi\gamma}}{dy}\right)_{KS} : \left(\frac{d\sigma^{\chi_{c2}}_{J/\psi\gamma}}{dy}\right)_{KS} = 1 : 0.1 : 0.4\,. \tag{4.4}$$

They are not affected by smaller x' or nonlinear effects in this model. As the normalization point we took the contribution of the $\chi_c(0^+)$ meson CEP as was done in Ref. [10].

In Table I we also presented results with the linear Kutak-Staśto model based on the unified BFKL-DGLAP framework and the nonlinear one based on the Balitsky-Kovchegov equation [31]. It turned out that incorporation of the nonlinear effects responsible for the gluon recombination in this model reduces the $\chi_c(J^+)$ CEP cross sections by 30-50 %. We see that the nonlinear effects play a crucial role in diffractive quarkonia production effectively decreasing the characteristic values of x' (controlled by ξ). However, reliable predictions including the nonlinear effects require the exact knowledge of the triple Pomeron vertex at NLLx accuracy, which is yet unknown.

It is also interesting to compare the diffractive production of χ_c states at different energies. As an example, in Table II we present the integrated (over full phase space) cross sections of $\chi_c(0^+,1^+,2^+)$ production at RHIC, Tevatron and LHC energies. The results show similar energy behavior of the diffractive cross section for different UGDFs as well as for different χ_c states.

Finally, let us turn to differential distributions. In Fig. 4 we show the differential cross section $d\sigma/dy$ in rapidity y for all χ_c states. In this figure and in the following, all helicity contributions for $\chi_c(1^+,2^+)$ CEP are taken into account. Here and below we show only bare CEP cross sections for GBW, KS and KMR UGDFs. In the last case, we present the results computed with the HKRS cut-off parameter $0.72\,\mathrm{GeV}^2$ [10]. We see that the shape of the

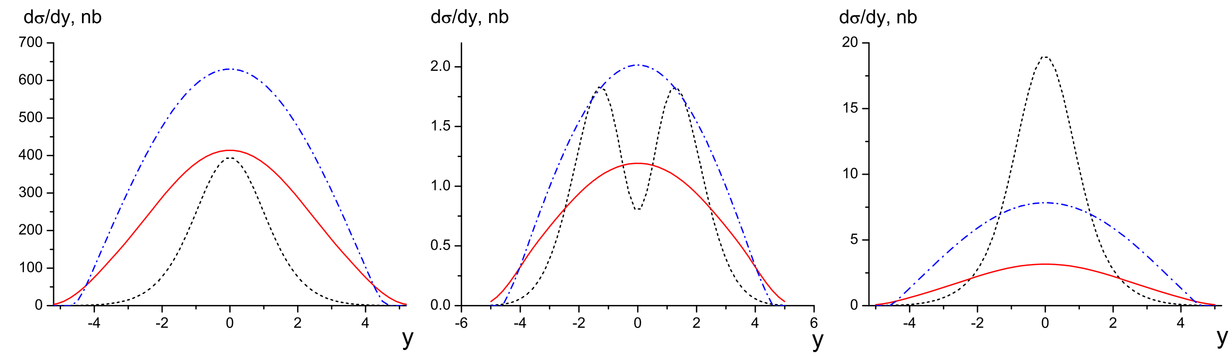

FIG. 4: *Distributions $d\sigma_{\chi_c}/dy$ in rapidity of $\chi_c(0^+)$ (left panel), $\chi_c(1^+)$ (middle panel) and $\chi_c(2^+)$ (right panel) mesons for different UGDFs at the Tevatron energy $\sqrt{s}$ = 1.96 TeV. The dash-dotted line corresponds to the KS UGDF [31] in the symmetrical "sqrt"-prescription with $\xi = 0.3$, solid line – KMR UGDF [13] with $R_g = 1.3$, $(Q_t^{cut})^2 = 0.72\,GeV^2$ and GRV94HO PDF [32], and short-dashed line represents result with the GBW UGDF [30] ($\xi = 0.3$). Absorption effects are not included here.*

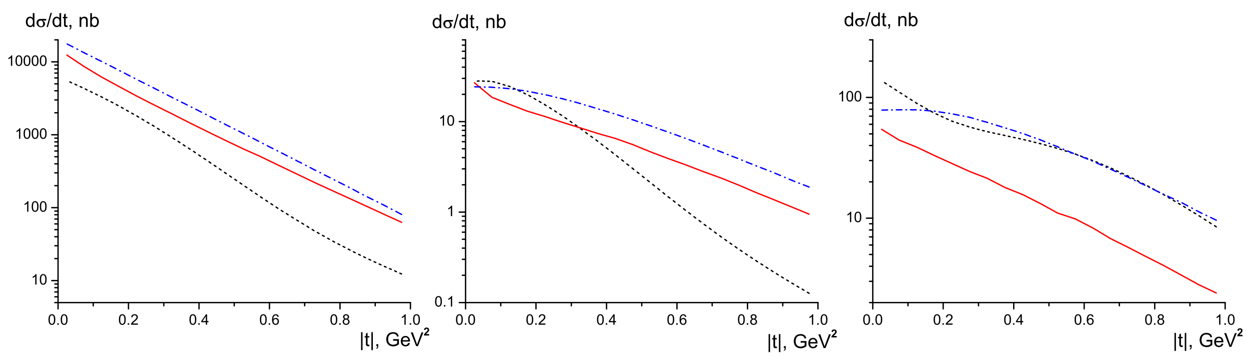

FIG. 5: *Distribution in $t_{1,2}$ of $\chi_c(0^+)$ (left panel), $\chi_c(1^+)$ (middle panel) and $\chi_c(2^+)$ (right panel) for meson CEP for different UGDFs. The meaning of curves here is the same as in Fig. 4.*

curves is rather similar, however, they have substantially different maxima. The biggest cross section for the $\chi_c(0^+, 2^+)$ states is obtained with the KS UGDF, whereas for $\chi_c(1^+)$ the KS and KMR UGDFs give quite similar cross sections.

In Fig. 5 we present corresponding distributions in $t = t_1$ or $t = t_2$ (identical), again for different UGDFs. Except of normalisation the shapes are rather similar. This is because of the t_1 and t_2 dependencies of form factors (describing the off-diagonal effect) are taken the same for different UGDFs.

In Fig. 6 we show the correlation function $d\sigma/d\Phi$ in relative azimuthal angle Φ between outgoing protons for different χ_c states. The shapes of the distributions are somewhat dependent on UGDFs. It is interesting to note here that the KS and KMR UGDFs lead to very similar angular dependence of $d\sigma/d\Phi$ for all χ_c states. In the case when energy resolution is not enough to separate contributions from different states of χ_c ($\chi_c(0^+)$, $\chi_c(1^+)$, $\chi_c(2^+)$), which seems to be the case for Tevatron, the distribution in relative azimuthal angle may, at least in principle, be helpful.

The fact that the angular distributions are not simple functions (like $\sin\Phi$, $\cos\Phi$) of the relative azimuthal angle between outgoing nucleons is due to the loop integral in Eq. (2.1)

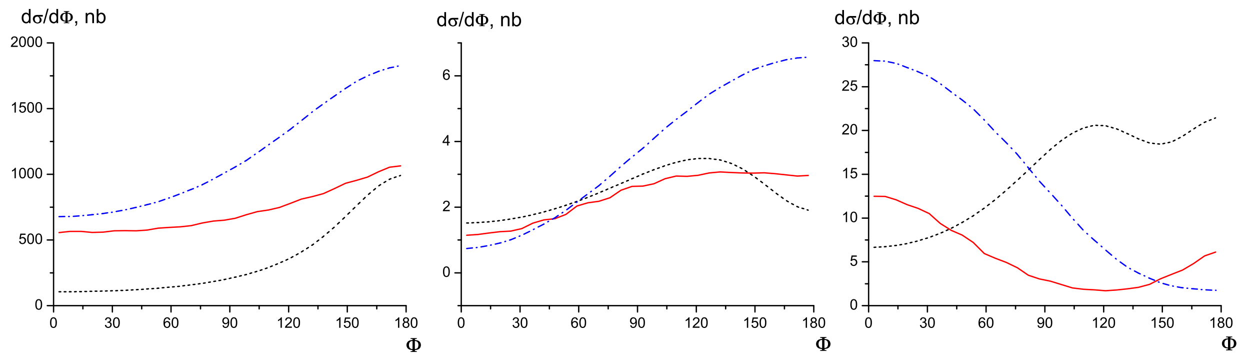

FIG. 6: *Distribution in relative azimuthal angle* Φ *between outgoing protons for* $\chi_c(0^+)$ *(left panel),* $\chi_c(1^+)$ *(middle panel) and* $\chi_c(2^+)$ *(right panel) meson CEP for different UGDFs. The meaning of curves here is the same as in Fig. 4.*

which destroys the dependence one would obtain with single fusion of well defined (spin, parity) objects (mesons or reggeons) [6].

V. CONCLUSIONS AND DISCUSSION

Our results can be summarized as follows:

We have derived the QCD amplitude for central exclusive production of tensor $\chi_c(2^+)$ meson. This amplitude vanishes in the forward limit of outgoing protons, as demanded by the $J_z = 0$ selection rule. Our numerical results show the importance of non-forward corrections, including all polarisation states of $\chi_c(2^+)$ and nonperturbative contributions to the $\chi_c(2^+)$ CEP. Inclusion of all the ingredients leads to a noticeable contribution of the $\chi_c(2^+)$ meson in the observable radiative decay channel depending on UGDF. We have observed the importance of the $\lambda = 0$ state $\chi_c(1^+)$ CEP and $\lambda = 0, \pm 1$ states for $\chi_c(2^+)$ CEP at $y \approx 0$, whereas the total CEP cross section is dominated by maximal helicity contributions.

The main contribution to diffractive charmonium production comes from small gluon transverse momenta $Q_t < 1\,\mathrm{GeV}$ leading to quite substantial sensitivity of the corresponding cross section on the infrared cut-off in perturbatively modeled KMR UGDF. Alternatively one could use UGDFs like Kutak-Staśto and GBW models, which by construction can be used for any values of the gluon transverse momenta.

We have tested the symmetrical prescription for off-diagonal UGDFs, following from positivity constraints and incorporating x, q_t dependence of both participating gluons, against the present CDF experimental data. A rather good quantitative agreement with the CDF data on charmonium CEP in the radiative decay channel is achieved with the nonlinear Kutak-Staśto UGDF model giving the cross section $d\sigma_{obs}(J/\psi\gamma)/dy(y=0) \simeq 0.6$ nb without imposing extra normalisation conditions beyond the QCD framework. Such a description is achieved by incorporating very soft screening gluons with $x' \sim 0.1 \cdot q_{0,t}/\sqrt{s}$. We have also calculated total cross sections of χ_c CEP at different energies (RHIC, Tevatron and LHC), as well as differential distributions in three phase space variables y, t, Φ.

Overall theoretical uncertainty of the QCD mechanism under consideration is rather high but hard to estimate due to large unknown nonperturbative contributions coming into the game and not well known higher-order QCD corrections to the hard subprocess

$g^*g^* \to \chi_c$ (especially, in the axial-vector case). Also, absorptive corrections may depend on UGDF used in the calculation, and there is no reliable estimation of such a sensitivity in literature. In the present paper we kept the strategy to study different distinct options and analyze the sensitivity of the final results with respect to the UGDFs choice, prescriptions for skewed UGDFs, nonperturbative cut-off parameter and characteristic x' variations, etc. Then a comparison with experimental data would allow to select the most reliable option. However, we observe a variety of such "good" options, namely, description of the data (with, however, pretty large theoretical uncertainties related, in particular, with unknown NLO corrections) can be, in principle, achieved for a few UGDFs (GBW, KS and KMR UGDFs, see Table I). Each of them pick up some essential QCD dynamics. Further constraints can, in principle, be settled by experimental measurements of separate $\chi_c(J^+)$ contributions, the energy dependence of the cross section and the shapes of differential distributions.

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